What QCD sum rules tell about the rho meson[*]

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Abstract

Using a simple parametrization of Breit-Wigner type for the hadronic side of the QCD sum rule for ρ mesons in vacuum as well as in a nuclear medium we explore the range of values for the mass and the width of the ρ meson which are compatible with the operator product expansion.

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I. INTRODUCTION

Recently the behavior of vector mesons in a nuclear surrounding became a lively debated issue. This was especially triggered by the new CERES experiments for S-Au and Pb-Au collisions which show an enhancement of the dilepton yield for invariant masses somewhat below the vacuum mass of the ρ meson [1–3]. As argued some years ago by Brown and Rho this enhancement might be due to the restoration of chiral symmetry [4]. In their approach the masses of the vector mesons scale with the quark condensate, i.e. drop with rising baryonic density. Thus the ρ peak in the dilepton spectrum ought to be shifted to lower invariant mass which indeed might be an explanation for the observed enhancement of the dilepton yield in that region [5–7]. However, also other scenarios utilizing the idea of chiral symmetry restoration are possible which predict a rising ρ mass based on the effect that the ρ becomes degenerate with its chiral partner, the $a1$ meson [8].

On the other hand, some calculations based on purely hadronic models gave rise to an alternative picture. Due to modifications of the pions in nuclear medium which at least in vacuum form the most important decay channel of the ρ meson the spectral function of the ρ might be drastically changed [9–11]. In addition, collisions of the vector meson with nucleons from the Fermi sea also influence the spectral function of the former [12–15]. Both pion modifications and collisions with nucleons at least lead to a broadening of the ρ peak, if not to completely new structures. Hence the enhancement in the dilepton yield might also be explained within a purely hadronic scenario where a lot of strength is shifted to lower invariant mass if the ρ peak becomes much broader basically without changing its pole position [13].

Qualitatively the Brown-Rho scenario was supported by QCD sum rule analysis which showed a dropping of the ρ mass, *if* the spectral function of the ρ is simply modeled by a δ-function [16–18]. On the other hand, the hadronic models cited above seem to indicate that this approximation might no longer be appropriate if the vector meson is placed in a medium e.g. with the saturation density of nuclear matter. A more sophisticated ansatz for the spectral function of the ρ meson including pion corrections caused by the nuclear surrounding was explored in [19,20]. Inserting this spectral function in the hadronic part of the QCD sum rule basically the same dropping of the ρ mass as from the simple pole ansatz was extracted. However, in [14], where also ρ-nucleon scattering was taken into account, no mass shift but only a large peak broadening was found. It was shown there that even then agreement between the left and the right hand side of the sum rule could be obtained. This finding clearly casts some doubt on the quantitative predictive power of the QCD sum rule analysis. It seems that only after one has chosen his favorite hadronic model the sum rule can tell whether this model is in agreement with the operator product expansion of QCD.

The purpose of the present paper is to explore in a systematic and as far as possible model independent way which spectral functions for the ρ meson actually are compatible with QCD sum rules. To do so we choose a simple Breit-Wigner parametrization for the spectral function treating the pole position and the on-shell width as toy parameters. Varying these parameters we calculate the difference of the left and right hand side of the sum rule. According to the finding of the two groups cited above [20,14] we do not expect to find one single set of reasonable parameters but at least a line or a whole band of parameter pairs. Of course, sophisticated hadronic models yield much more complicated spectral functions than

the one we will use here. However, since the spectral function appears only in an integral in the sum rule (cf. eq. (14) below) we expect that the sum rule is not sensitive to the details of the modeling of the spectral function, but only to its gross features. Therefore we believe that our analysis using a simple Breit-Wigner parametrization is also applicable to more sophisticated hadronic models.

In the next section we briefly recapitulate the derivation of the QCD sum rule for the ρ meson in vacuum as well as in a nuclear medium. In section III we present our parametrization for the spectral function. Numerical results are given in section IV. Finally we summarize our results in V.

II. OPERATOR PRODUCT EXPANSION

We start with the time ordered current-current correlator

$$\Pi_{\mu\nu}(q) = i \int d^4x \, e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle \tag{1}$$

where j_μ is the electromagnetic current of the ρ meson,

$$j_\mu = \frac{1}{2} \left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d \right) . \tag{2}$$

In the following we will write down all formulae for the case of finite nuclear density ρ_N. The vacuum case is recovered by simply putting ρ_N to zero.

At finite baryo-chemical potential Lorentz invariance is broken. From q^μ and the baryonic current one can construct two independent projectors $L_{\mu\nu}(q)$ and $T_{\mu\nu}(q)$ which both still satisfy current conservation $q^\mu L_{\mu\nu}(q) = q^\mu T_{\mu\nu}(q) = 0$ (cf. e.g. [21]). Hence the correlator can be decomposed in the following way:

$$\Pi_{\mu\nu} = \Pi^T T_{\mu\nu} + \Pi^L L_{\mu\nu} . \tag{3}$$

The scalar functions Π^T and Π^L in general depend on q^2 and $\vec{q}^2$ where $\vec{q}$ is defined in the rest frame of the nuclear medium. For simplicity we restrict ourselves here to the case of vanishing $\vec{q}$, i.e. where the ρ meson is at rest relative to the baryonic current. In this case Π^T and Π^L become equal,

$$\Pi^T(q^2, \vec{q}^2 = 0) = \Pi^L(q^2, \vec{q}^2 = 0) =: \Pi(q^2) . \tag{4}$$

Actually, most of the calculations concerning QCD sum rules for vector mesons in medium were performed for this special case, e.g. [16,17,20,14]. Only recently there appeared growing interest in the explicit $\vec{q}$ dependence of the ρ spectral function [12,13,15]. The traditional approach to QCD sum rules for vector mesons, i.e. the pole + continuum ansatz, was recently generalized to finite $\vec{q}$ by Lee [22]. A generalization of the approach presented here to finite $\vec{q}$ is in progress.

For large space like four-momenta the correlator (1) and thus the scalar function Π can be calculated from the operator product expansion (OPE) of QCD [23]. Utilizing a subtracted dispersion relation one can make contact with the time like region by (see e.g. [14])

$$\Pi(Q^2) = \Pi(0) + cQ^2 + \frac{Q^4}{\pi} \int\limits_0^\infty ds \, \frac{\mathrm{Im}\Pi(s)}{s^2\,(s + Q^2 - i\epsilon)} \tag{5}$$

where we have introduced $Q^2 = -q^2$ and c is a subtraction constant. Now the basic idea of QCD sum rules is to calculate the l.h.s. of (5) by the OPE and the r.h.s. by a hadronic spectral function.

Following [16,24] the OPE for the dimensionless quantity

$$R(Q^2) = \frac{\Pi(Q^2)}{Q^2} \tag{6}$$

including condensates up to dimension 6 is given by

$$R(Q^2) = SC + \frac{m_N}{4Q^4} A_2 \rho_N - \frac{5m_N^3}{12Q^6} A_4 \rho_N \tag{7}$$

where SC denotes the contribution from the scalar condensates,

$$SC = -\frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right)\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{1}{Q^4}m_q\langle\bar{q}q\rangle + \frac{1}{24Q^4}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \frac{112}{81Q^6}\pi\alpha_s\kappa\,\langle\bar{q}q\rangle^2\,. \tag{8}$$

The last two contributions to (7) arise from the non scalar condensates $\langle \mathcal{S}\bar{q}\gamma_\mu D_\nu q\rangle$ and $\langle \mathcal{S}\bar{q}\gamma_\mu D_\nu D_\sigma D_\tau q\rangle$ (see [16,24] for details). The medium dependence of the condensates is taken into account in leading order in the baryon density ρ_N [16] (for a review on QCD sum rules in nuclear matter cf. [25]). For the scalar condensates this yields

$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_{\mathrm{vac}} + \frac{\sigma_N}{2m_q}\rho_N \tag{9}$$

and

$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\mathrm{vac}} - \frac{8}{9}m_N^{(0)}\rho_N\,. \tag{10}$$

We take the following numerical values (cf. [14]) for
– the strong coupling constant, $\alpha_s = 0.36$,
– the current quark mass, $m_q = 7\,\mathrm{MeV}$,
– the vacuum two quark condensate, $\langle\bar{q}q\rangle_{\mathrm{vac}} = (-250\,\mathrm{MeV})^3$,
– the vacuum gluon condensate, $\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\mathrm{vac}} = 1.2\cdot 10^{-2}\,\mathrm{GeV}^4$,
– the nucleon sigma term, $\sigma_N = 45\,\mathrm{MeV}$,
– and the nucleon mass in the chiral limit, $m_N^{(0)} = 750\,\mathrm{MeV}$.
The factors A_2 and A_4 are determined from the quark parton distributions. We use the values [16] $A_2 = 0.9$ and $A_4 = 0.12$. Finally κ parametrizes the deviation of the four quark condensate from the product of two quark condensates, i.e. from the Hartree approximation. This value is still not very well determined. Therefore we will choose different values for κ to check the sensitivity of our results. Following [16,14,26], respectively, we take

$$\kappa = 1,\ 2.36,\ 6 \tag{11}$$

4

and neglect a possible ρ_N dependence of κ as is done usually in such studies [16–18,20,14]. This is, however, a point that clearly deserves more attention since model studies [27] show that in the case of finite density the RPA excitations out of positive energy states counteract the effects of the particle-antiparticle excitations.

It is important to note that we have not taken into account all condensates up to dimension 6. Especially, we have neglected higher twist operators since they are smaller than the twist two contributions [28,29] and their determination from the experiment is not very accurate yet. Their influence on the OPE for the vector mesons in medium is discussed in [17,22].

The convergence of the OPE can be improved by applying a Borel transformation to (5). The final result is (see e.g. [30] for details about the Borel transformation)

$$
\frac{1}{\pi M^2} \int_0^\infty ds\, \mathrm{Im} R_{\mathrm{HAD}}(s)\, e^{-s/M^2} = \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right)
$$
$$
+ \frac{1}{M^4} m_q \langle \bar{q}q \rangle + \frac{1}{24 M^4}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{1}{4 M^4} m_N A_2 \rho_N
$$
$$
- \frac{56}{81 M^6}\pi \alpha_s \kappa \langle \bar{q}q \rangle^2 - \frac{5}{24 M^6} m_N^3 A_4 \rho_N \tag{12}
$$

where M denotes the Borel mass and we have introduced the label HAD to stress that $\mathrm{Im}R_{\mathrm{HAD}}$ is calculated from a hadronic model. In addition, we have absorbed the subtraction constant $\Pi(0)$ appearing in (5) into $\mathrm{Im}R_{\mathrm{HAD}}$. It is given by the Landau damping contribution [16] presented in the following section.

III. PARAMETRIZATION OF THE SPECTRAL FUNCTION

Following previous works [16,18,14] we decompose $\mathrm{Im}R_{\mathrm{HAD}}$ in three parts, the contribution from the ρ meson, the continuum part, and the Landau damping contribution:

$$
\mathrm{Im}R_{\mathrm{HAD}}(s) = \pi F \frac{S(s)}{s}\Theta(s_0 - s) + \frac{1}{8\pi}\left(1 + \frac{\alpha_s}{\pi}\right)\Theta(s - s_0) + \delta(s)\frac{\pi}{4}\rho_N \frac{1}{\sqrt{k_F^2 + m_N^2}}. \tag{13}
$$

The sum rule we will examine in the following is obtained from (12) by inserting the decomposition (13) and taking the continuum and Landau damping contributions to the r.h.s.,

$$
\frac{1}{M^2}\int_0^{s_0} ds\, F \frac{S(s)}{s} e^{-s/M^2} = \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right)\left(1 - e^{-s_0/M^2}\right) - \frac{1}{4M^2}\rho_N \frac{1}{\sqrt{k_F^2 + m_N^2}}
$$
$$
+ \frac{1}{M^4} m_q \langle \bar{q}q \rangle + \frac{1}{24 M^4}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{1}{4 M^4} m_N A_2 \rho_N
$$
$$
- \frac{56}{81 M^6}\pi \alpha_s \kappa \langle \bar{q}q \rangle^2 - \frac{5}{24 M^6} m_N^3 A_4 \rho_N. \tag{14}
$$

If strict vector meson dominance (VMD) is applied then S would be the spectral function of the ρ meson (see e.g. [20]). Since we are examining only a parametrization of S in the

following we do not have to specify whether we are using VMD or not. Anyway we will refer to S as a spectral function.

The key issue now is the choice for the spectral function S. Clearly in vacuum the spectral function shows a single peak structure which is, of course, at the position of the free ρ mass. In a nuclear medium we have no guideline from experiment how the spectral function might look like. However, examining the hadronic functions presented in the literature (e.g. [13,14]) we also find single peak structures, possibly somewhat shifted as compared to the vacuum case. Thus we will also model our spectral function such that it shows one peak at invariant mass m_ρ with width γ. We will treat these two quantities as free parameters to figure out which combinations of them are actually compatible with OPE.

We choose the following parametrization

$$S(s) = \frac{1}{\pi} \frac{\sqrt{s}\,\Gamma(s)}{(s - m_\rho^2)^2 + s\,(\Gamma(s))^2}, \tag{15}$$

i.e. we especially neglect a possible s dependence of the mass parameter.

Since the integral appearing in (14) is obviously sensitive to the behavior of $S(s)$ for small values of s it is important to model the threshold behavior of Γ in a physically reasonable way. In vacuum the spectral function is dominated by the decay of the ρ into two pions. Taking into account the phase space as well as the derivative coupling of the ρ-π-π vertex we find the threshold behavior

$$\Gamma_{\text{vac}}(s) \sim (s - 4m_\pi^2)^{3/2}\,\Theta(s - 4m_\pi^2). \tag{16}$$

Keeping things as simple as possible we use for the vacuum case

$$\Gamma_{\text{vac}}(s) = \gamma \left(\frac{1 - \dfrac{4m_\pi^2}{s}}{1 - \dfrac{4m_\pi^2}{m_\rho^2}} \right)^{3/2} \Theta(s - 4m_\pi^2) \tag{17}$$

with the constant γ being the on-shell width of the vector meson. For a more sophisticated parametrization of the spectral function in connection with QCD sum rules in vacuum we refer to [31].

For finite nuclear density not only the decay into pions but also the scattering with nucleons influences the spectral function. The latter effect also contributes below the two pion threshold. If the Fermi motion of the nucleons is neglected the threshold for the spectral function of a ρ meson *at rest* is now given by the mass of one pion since the lightest pair of particles which can be formed in the ρ-nucleon collision is a nucleon and a pion. The threshold behavior is dominated by the lowest possible partial wave. Without any additional constraint from the intermediate state formed in the ρ-nucleon collision we assume it to be an s-wave state. Hence we get

$$\Gamma_{\text{med}}(s) \sim (s - m_\pi^2)^{1/2}\,\Theta(s - m_\pi^2) \tag{18}$$

and thus

6

$$\Gamma_{\mathrm{med}}(s) = \gamma \left(\frac{1 - \frac{m_\pi^2}{s}}{1 - \frac{m_\pi^2}{m_\rho^2}} \right)^{1/2} \Theta(s - m_\pi^2). \tag{19}$$

If one wants to study the sum rule for very low densities both thresholds (one and two pion masses) are relevant. In this case one has to think about a description how to smoothly switch on the one pion threshold with increasing density. In the following, however, we will concentrate on the case of saturation density of nuclear matter. Here, if the width is chosen large enough, there is already a lot of strength in the spectral function below the two pion threshold. Hence the two pion threshold does not influence the gross features of the spectral function for the densities we are interested in.

We note that there are remarkable exceptions from the single peak structure by mentioning two examples: First, if the spectral function of the ρ in medium is modeled by a pion loop where the pions are dressed due to medium effects a second sharp peak at low invariant mass shows up besides the usual broad ρ peak [9,10,20]. Second, a two pole structure with two maxima appears if collisions with nucleons from the Fermi sea are taken into account for the calculation of the ρ spectral function. This additional pole is basically generated by the coupling of the nucleon-ρ system to the $N(1520)$ resonance [15].[1] Such spectral functions are clearly not covered by our toy spectral function. Therefore strictly speaking, our parametrization is still model dependent. However we cover a large class of models with our ansatz since the sum rule (14) is not sensitive to the details of the spectral function but only to its gross features like the threshold behavior.

IV. EXAMINING THE SUM RULE

Having set up our formalism we can ask which values for mass m_ρ and width γ are compatible with OPE, i.e. which set of parameters can be inserted in (15) such that the sum rule (14) is satisfied. Of course the sum rule is not valid for arbitrary values of the Borel mass M. If M becomes too small the expansion in orders of $1/M^2$ breaks down. On the other hand for very large values of M the contribution from perturbative QCD completely dominates the sum rule. In this case we are no longer sensitive to the parameters we are interested in. In [26,18] the following recipe was suggested to determine a reasonable Borel window: The minimal M is determined such that the terms of order $o(1/M^6)$ on the r.h.s. of (12) contribute no more than 10% to the total value of the r.h.s. For the maximal M we require that the continuum part is not larger than the contribution of the spectral function to the l.h.s. of (12), i.e.

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[1]Actually, the two peak structure shows up if the spectral function is calculated in lowest order in the nuclear density. If higher order effects are taken into account the second peak is washed out to a large extent (see [15] for details).

$$\int\limits_0^\infty ds \, \frac{1}{8\pi} \left(1 + \frac{\alpha_s}{\pi}\right) \Theta(s - s_0) \, e^{-s/M^2} \leq \int\limits_0^\infty ds \, \pi F \frac{S(s)}{s} \Theta(s_0 - s) \, e^{-s/M^2} . \tag{20}$$

For given values of m_ρ, γ, F, and s_0 we calculate the relative deviation of the l.h.s. from the r.h.s. of (14) and average these deviations over the Borel window defined above, schematically

$$d = \int\limits_{M_{\text{min}}^2}^{M_{\text{max}}^2} d(M^2) \, |1 - \text{l.h.s.}/\text{r.h.s.}| \, /\Delta M^2 \tag{21}$$

with

$$\Delta M^2 = M_{\text{max}}^2 - M_{\text{min}}^2 . \tag{22}$$

Since we are mainly interested in the ranges of reasonable values for mass and width we still have to determine F and s_0. We use the following strategy: For given values of m_ρ, γ, and s_0 we fix F by the finite energy sum rule [16],

$$\int\limits_0^{s_0} ds \, \text{Im} R_{\text{HAD}}(s) = \int\limits_0^{s_0} ds \, \frac{1}{8\pi} \left(1 + \frac{\alpha_s}{\pi}\right) , \tag{23}$$

hence

$$F = \frac{s_0 \frac{1}{8\pi^2} \left(1 + \frac{\alpha_s}{\pi}\right) - \frac{1}{4}\rho_N \frac{1}{\sqrt{k_F^2 + m_N^2}}}{\int\limits_0^{s_0} ds \, \frac{S(s)}{s}} . \tag{24}$$

For given values of m_ρ and γ, s_0 is determined by the requirement that d in (21) ought to be minimal.

Now we require two criteria for a pair of parameters m_ρ and γ to hold. First, the difference d as given in (21) should be reasonably small. In the following we will explore the region of parameter pairs which fulfill $d \leq 0.2\%$ and $d \leq 1\%$, respectively. Second, the Borel window as defined in (22) should be reasonably large. In [26] a window of $1.4\,\text{GeV}^2$ was found for the vacuum case in the traditional sum rule approach (vanishing width). As shown in [18] the Borel window shrinks with increasing nuclear density. At nuclear saturation density the extension of the Borel window was found to be $0.9\,\text{GeV}^2$. For vanishing width these are, of course, optimal values and not lower limits. In our calculations we require the window ΔM^2 to be larger than $0.9\,\text{GeV}^2$ for the vacuum case; for nuclear saturation density we take a value of $0.6\,\text{GeV}^2$ as a lower limit for ΔM^2.

The parameter pairs which fulfill both criteria are plotted in figs. 1,2 for vacuum and nuclear saturation density for the values for κ as given in (11). To get an idea how the two criteria influence the final result we have disentangled them in fig. 1 for the vacuum case and $\kappa = 2.36$. If no constraint on the width of the Borel window is imposed the left one of

the dashed lines is replaced by the dotted line. (The other lines stay the same.) Obviously the window criterion gives an important constraint by cutting off low values of the mass. If the lower limit for ΔM^2 is increased the corresponding line (the left dashed one) would move further to the right. In all the other plots of figs. 1,2 the line which is most to the left is determined by the window criterion.

For the strict requirement $d \leq 0.2\%$ and for $\kappa = 1$ we find only a small band of parameters for the vacuum case and no parameters for the case of nuclear saturation density. Note, however, that Hatsuda et $al.$ [16,24,17] who used $\kappa = 1$ took a somewhat larger value for the two quark condensate. This would presumably yield a picture similar to our case $\kappa = 2.36$, at least for vacuum (fig. 1). Actually, only for the choice $\kappa = 2.36$ the experimental values for the vacuum case, $m_\rho = 0.77 \, \mathrm{GeV}$ and $\gamma = 0.15 \, \mathrm{GeV}$, can be found inside the parameter range which is in accordance with $d \leq 0.2\%$. This is however not surprising since $\kappa = 2.36$ together with the values for the condensates we have used here are the result of an optimization to fulfill the sum rule in vacuum reasonably well [14]. The authors who have used the values $\kappa = 1$, 6 also use different values for the other condensates. Clearly, it would be of interest to see how a variation of the other condensate values influences our results. For simplicity we have restricted ourselves here to a variation of κ, only.

The values for s_0, F, and the Borel window, of course, differ for each pair of parameters m_ρ and γ. To get some idea about their magnitudes we have listed in table I their values for some selected parameter pairs. One can clearly observe the tendency of the continuum threshold s_0 to decrease with increasing density. This is in agreement with the results from the traditional sum rule approach [16–18] and was used as an input in the analysis of [14]. To demonstrate that also the values for F as determined from the finite energy sum rule (23) are in a reasonable range we take $F = 0.01 \, \mathrm{GeV}^4$ as derived for the vacuum case with $\kappa = 2.36$ for the physical values $m_\rho = 0.77 \, \mathrm{GeV}$ and $\gamma = 0.15 \, \mathrm{GeV}$ (cf. table I). Assuming vector meson dominance F is indeed given by $F \approx m_\rho^4/g_\rho^2 \approx 0.01 \, \mathrm{GeV}^4$ [24] where g_ρ is the ρ-π-π coupling constant. Also F decreases with increasing density reflecting the redistribution of ρ-strength in the medium.

The results in figs. 1,2 are in qualitative agreement with previous work for all values of κ. For vanishing width the mass decreases with increasing density. This is the result of the traditional sum rule approach. However, with increasing width the window of "good" values for m_ρ is shifted to the right, i.e. to larger values. For finite density this shift becomes even stronger. This confirms the finding that for a small value of the width the mass drops in medium [20], while for a large value of the width the mass might stay constant [14].

With rising width the window for reasonable masses considerably grows if the weaker criterion $d \leq 1\%$ is applied. Therefore, the sum rule approach loses more and more of its predictive power. For e.g. $\kappa = 2.36$ and $d \leq 1\%$ we find that for small width the mass can be determined reasonably well within an uncertainty of about $0.1 \, \mathrm{GeV}$. For large width on the other hand the window of allowed values for the mass grows to $0.3 - 0.4 \, \mathrm{GeV}$. This in turn means that in any hadronic model that leads to a large width for the spectral function of the rho meson any agreement of the calculated mass with the QCD sum rule prediction is not a very strong statement. In any case our analysis shows that one cannot extract model independent informations about the properties of the ρ meson from the sum rules. As already mentioned, in all the figures the left one of the dashed lines is basically determined by the second criterion which requires that the Borel window is reasonably large. If this

criterion is removed the left one of the dashed lines would be placed much more to the left (cf. fig. 1 for $\kappa = 2.36$). Then the uncertainty in the determination of the mass for vanishing width would also be much larger than $0.1\,\text{GeV}$.

From the figures for different values of κ we find that the masses rise with growing κ. At first sight, one would tend to exclude the cases $\kappa = 1$, 6 for vacuum since the experimental values for mass and width are not inside the parameter range supported by the sum rule. This, however, does not necessarily mean that these values for κ ought to be discarded for finite density, too. Since κ might explicitly depend on the density the values we have used are not unrealistic a priori. Fig. 2 shows that even for vanishing width a κ which grows with the density can compensate for the negative mass shift otherwise caused by the density and restore the vacuum value of the mass. We conclude that the sum rule analysis is still plagued by a large uncertainty in the determination of the condensate values, especially concerning the four quark condensate.

V. SUMMARY

We have presented a systematic study of the QCD sum rule for the ρ meson to shed some light on the question which hadronic spectral functions for the ρ meson are compatible with the operator product expansion of QCD. Using a simple Breit-Wigner ansatz with the correct threshold behavior incorporated we explored the range of parameters for the ρ mass and on-shell width which satisfy the QCD sum rule. From the qualitative point of view the important message is that there is not only one pair of reasonable parameters. Quantitatively we find that for small (large) values of the mass also the width has to be small (large). Thus the lowest mass is given by the traditional calculations using a δ-function for the spectral function [16,17,26,18]. However, especially for large values of the width a large range of values for the mass would also be allowed. We conclude that specific properties of the ρ meson in a nuclear medium cannot be determined from QCD sum rules without specifying a hadronic model. Instead the sum rules only provide a (wide) constraint for reasonable hadronic models. Even then, the factorization of the four-quark condensate, i.e. the dependence of any result on the parameter κ and its possible density dependence, introduces an additional uncertainty.

Nevertheless the sum rules are useful, first, to provide a constraint for reasonable hadronic models or, second, to incorporate additional medium dependences of hadronic parameters (see e.g. [20]). We stress again, however, that the sum rule loses its predictive power with increasing width since the window for values of the mass supported by the sum rule drastically grows. For any further study concerning the comparison of a hadronic model with the OPE we suggest to quantify any statement about the agreement with the QCD sum rule. We advocate to use the criterion for a reasonably large Borel window as presented by Leinweber *et al.* [26,18] together with the deviation d presented here as a tool to check the compliance of any hadronic model with the OPE.

The work presented here is restricted to ρ mesons which are at rest in the nuclear medium. A generalization to finite momentum is in progress.

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	m_ρ [GeV]	γ [GeV]	s_0 [GeV^2]	F [GeV^4]	$M^2_{\min}$ [GeV^2]	$M^2_{\max}$ [GeV^2]	d
vacuum, $\kappa = 2.36$							
opt.	0.71	0.01	1.1	0.008	0.69	1.6	0.0011
exp.	0.77	0.15	1.2	0.01	0.69	1.8	0.0015
vacuum, $\kappa = 1$							
opt.	0.71	0.48	0.96	0.01	0.51	1.4	0.0013
opt.0	0.64	0.01	0.95	0.06	0.51	1.4	0.0070
exp.	0.77	0.15	1.3	0.01	0.51	1.9	0.014
vacuum, $\kappa = 6$							
opt.	0.90	0.08	1.7	0.02	0.95	2.5	0.0013
opt.0	0.88	0.01	1.6	0.02	0.95	2.4	0.0013
exp.	0.77	0.15	1.1	0.01	0.95	1.5	0.0097
nuclear saturation density, $\kappa = 2.36$							
opt.	0.66	0.22	0.88	0.006	0.54	1.3	0.00085
opt.0	0.57	0.01	0.78	0.003	0.54	1.1	0.0019
nuclear saturation density, $\kappa = 1$							
opt.	0.62	0.48	0.68	0.005	0.41	1.0	0.0035
opt.0	0.49	0.01	0.67	0.002	0.41	1.0	0.0092
nuclear saturation density, $\kappa = 6$							
opt.	0.74	0.01	1.2	0.009	0.74	1.7	0.0011

TABLE I. Values for s_0, F, $M^2_{\min}$, $M^2_{\max}$, and d for some selected parameter pairs m_ρ and γ. *opt.* denotes the optimal value (minimal d) of the whole parameter range. *opt.0* denotes the optimal value for the smallest width we have studied. *exp.* denotes the experimental values $m_\rho = 0.77\,\text{GeV}$ and $\gamma = 0.15\,\text{GeV}$.
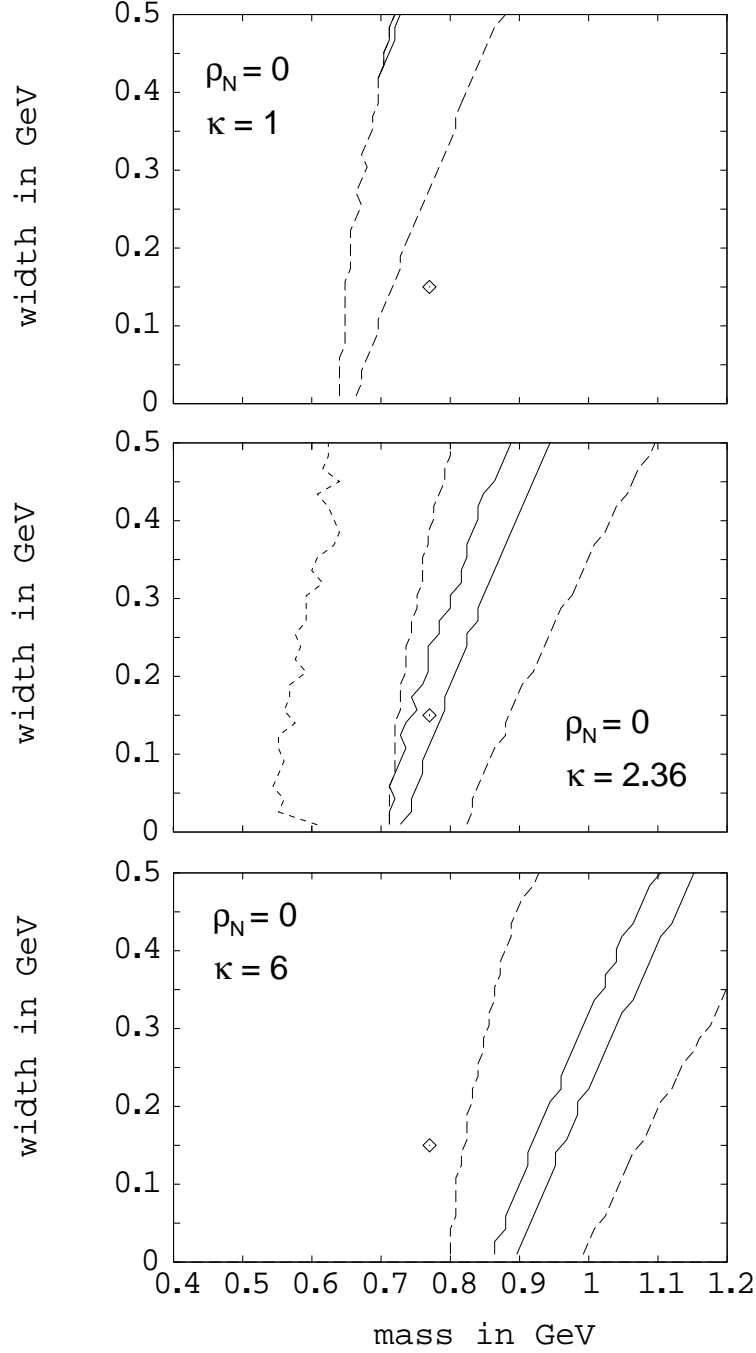
FIG. 1.   The width γ over the mass m_ρ for vacuum and for different values of κ. The full lines border the region of QCD sum rule allowed parameter pairs with $d \leq 0.2\%$ and $\Delta M^2 \geq 0.9\,\text{GeV}^2$, the dashed lines border the allowed region for $d \leq 1\%$ (same ΔM^2). The diamond marks mass and width of the free ρ meson. The leftmost (dotted) line in the middle picture ($\kappa = 2.36$) gives the lower boundary for the $d \leq 1\%$ criterion if no constraint on the Borel mass window is imposed.
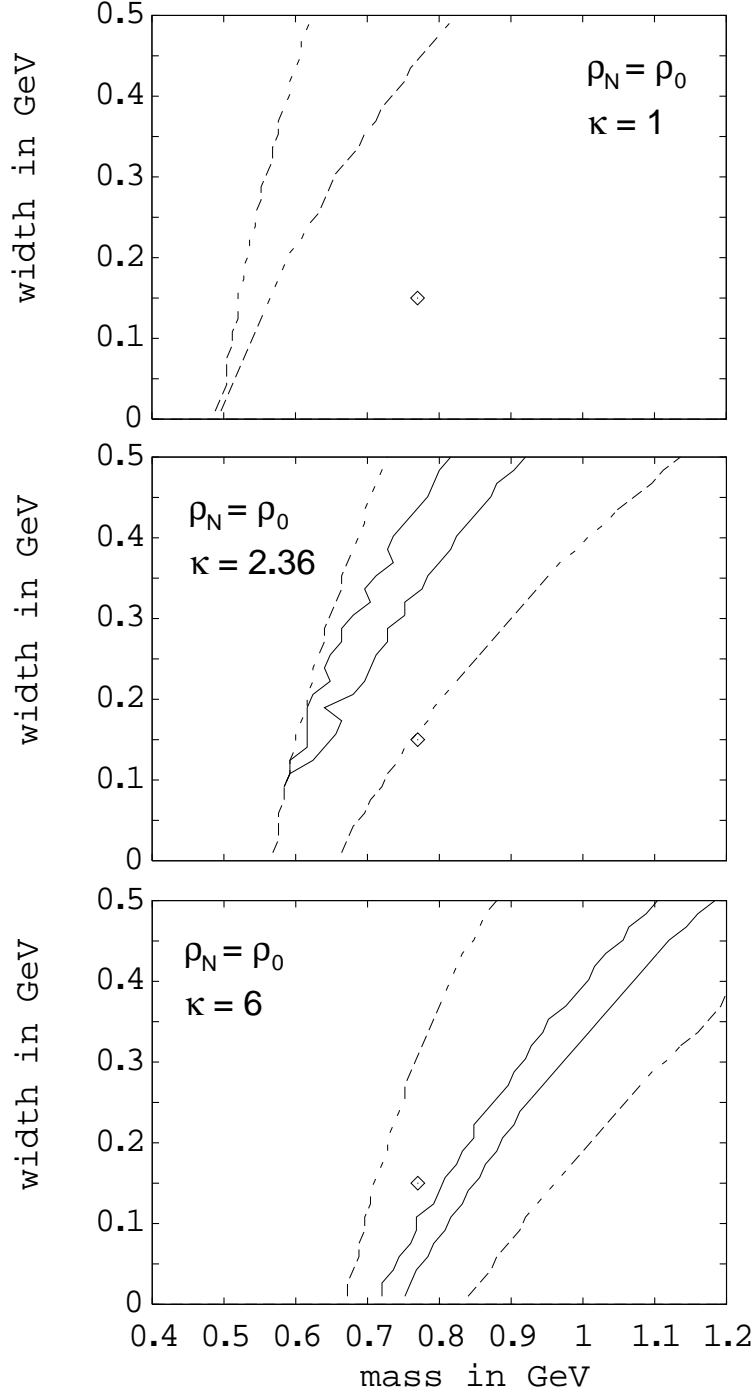
FIG. 2.   The width γ over the mass m_ρ for nuclear saturation density ρ_0 and for different values of κ. The full lines border the region of QCD sum rule allowed parameter pairs with $d \leq 0.2\%$ and $\Delta M^2 \geq 0.6\,\mathrm{GeV}^2$, the dashed lines border the allowed region for $d \leq 1\%$ (same ΔM^2). The diamond marks mass and width of the free ρ meson.